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Title: Rear-end Collision Events: Characterization of Impending Crashes.<br>Authors: $\quad$ Peter G. Martin, August L. Burgett<br>Affiliation: National Highway Traffic Safety Administration<br>NRD-12, Room 6220<br>400 7th Street SW<br>Washington DC 20590<br>Primary Author: Peter G. Martin<br>Email: peter.martin@nhtsa.dot.gov<br>Phone: (202) 366-5668<br>Rev. Date: October 10, 2001


#### Abstract

This paper demonstrates how a single data record (rather than several time-histories) may be used to characterize an impending two-car, rear-end collision in which a lead-vehicle and following-vehicle are initially separated by a range of $\mathrm{R}_{0}$. The characterization process assumes an idealized deceleration profile: a vehicle initially traveling at velocity $\mathrm{V}_{\mathrm{o}}$ decelerates instantaneously from zero to a constant level, $\mathrm{d}_{\mathrm{o}}$, at some brake application time, $\mathrm{t}_{\mathrm{b}}$. Therefore, a set of seven single-valued covariates ( $\mathrm{V}_{\mathrm{Fo}}, \mathrm{d}_{\mathrm{Fo}}, \mathrm{t}_{\mathrm{Fb}}, \mathrm{V}_{\mathrm{Lo}}, \mathrm{d}_{\mathrm{Lo}}, \mathrm{t}_{\mathrm{Lb}}$, and $\mathrm{R}_{\mathrm{o}}$ ) may be used to describe the actions of both vehicles. In driving simulator tests, the unit may be programmed to meet the ideal deceleration assumption. In naturalistic driving, however, the assumption rarely holds true. Nonetheless, many naturalistic braking events may be accurately represented by a carefully chosen set of covariates. The data reduction procedure described herein shows how the covariates are determined from just two measurements: following-vehicle velocity and inter-vehicle range. The validity of the process is demonstrated using data from the Iowa Driving Simulator, where the actual deceleration profiles of both vehicles are shown for comparison. The process is then applied to naturalistic driving data from a field operational test of an Intelligent (Adaptive) Cruise Control system.


## BACKGROUND

The rear-end crash type encompasses collisions that occur when the front of a following-vehicle strikes the rear of a lead-vehicle, with both traveling in the same lane. When an impending rear-end crash situation develops, a crash is almost always avoided. On average, a single driver will brake about 50,000 times per year to avoid crashing into a lead-vehicle. A rear-end crash occurs only about once every 2.5 million brake applications (Farber, 1991). Figure 1 illustrates the scale of braking events in terms of miles driven and shows the infrequency of a rear-end crash. Nonetheless, rear-end collisions comprise about $25 \%$ of all police-reported collisions in the United States, or approximately 1.8 million collisions per year. They also account for about 2,000 fatalities and over 800,000 injured occupants.


FIGURE 1 Hierarchy of Forward Crash Events

Many rear-end crashes may be avoided through the use of some sort of forward crash warning (FCW) system that signals a driver to brake. An FCW system typically involves a range-finding device (usually based on laser or radar technology) on the front of a host-vehicle that tracks a lead-vehicle. Together with a host-vehicle velocity signal (and perhaps an acceleration signal if the vehicle is instrumented with an accelerometer), the range signal is fed to an in-vehicle data processor. When a lead-vehicle is encountered, the processor analyzes the situation to determine whether a warning is warranted.

In an effort to develop an effective FCW, several studies of rear-end crash dynamics have been undertaken. Many of these studies have used driving simulators to examine how volunteer drivers react to collision warnings. For example, the University of Iowa's Iowa Driving Simulator has been used to study the effects of various rear-end crash warnings on driver behavior (McGehee and Brown, 1998). The authors found warnings to be most effective when headways are shortest. They also found warnings to be confusing or aggravating when they are issued too early, when drivers are already braking, and when drivers are being distracted.

FCW evaluations have also been based on test track experiments. The Crash Avoidance Metrics Partnership (CAMP) administered a series of controlled tests carried out at the General Motors Proving Grounds in Milford, MI (Kiefer et al, 1999). In this setting, volunteers driving a production vehicle modified with an FCW system were asked to follow a realistic-looking lead-vehicle that was really just a thermoformed plastic body. Like the driving simulator volunteers, the test track drivers found themselves in imminent crash situations when the lead-vehicle stopped suddenly. The situations often resulted in crashes with the plastic body.

Field operational tests (FOT's) - where "naturalistic" driving data are collected using modified production vehicles driven on city streets and highways - have been a part of the effort, too. In an FOT co-sponsored by the National Highway Traffic Safety Administration (NHTSA) and the University of Michigan's Transportation Research Institute (Fancher et al, 1998), a 12 -vehicle fleet of passenger cars was equipped with Intelligent (Adaptive) Cruise Control (ICC). An ICC system is similar to conventional cruise control, but it detects a lead-vehicle and will maintain a specified headway without driver interaction. An ICC-equipped vehicle will automatically slow down as it approaches a slower-traveling lead-vehicle. Though the system evaluated in the FOT did not include a driver warning, it was equipped with enough instrumentation to examine many driver tendencies.

These three studies - and others like them - furnish valuable insight into the behavior of drivers and the full range of conditions that must be considered when measuring the effectiveness of any warning system. However, each provides only a slice of the driving experience and studies based solely on each slice are incomplete. In general, naturalistic data are considered genuine because they are taken from an actual highway experience, not contrived using a test track or simulator. Naturalistic data are made up mostly of everyday driving events - those shown on the lower end of the Fig. 1 scale. But imminent crash situations seldom occur in naturalistic driving - both with and without an FCW system - making it difficult to evaluate an FCW system for the type of events that it is designed to avoid. On the other hand, test track and driving simulator data are made up almost entirely of rare "conflict" events, which are shown on the upper end of the Fig. 1 scale.

Presented herein is a method to characterize braking events from all types of data sources. It reduces the several time-histories that describe the event to a set of seven single-valued covariates. Subsequently, a database of events may be assembled from various sources so that a comprehensive assessment of driver behavior and crash warning systems may be carried out.

## CHALLENGES

The technology and instrumentation needed to implement a forward collision warning (FCW) system has improved greatly in recent years, and continues to progress. Range-finding devices are becoming more mature, and the use of global position satellites (GPS) is making inroads into advanced FCW systems. Notwithstanding these improvements, knowledge of driver behavior in a rear-end crash situation is still lacking. The characterization method presented herein provides a means to better understand driver behavior and meet the challenges associated with an FCW system implementation.

One such challenge is that of establishing appropriate warning thresholds for an FCW system. Most efforts to date have been based on a limited data set. For example, LeBlanc et al (2001) regressed headway, speeds, and
decelerations gathered in CAMP tests (the test track data mentioned earlier) to determine the thresholds for deceleration and reaction time. The characterization method presented herein provides a means to expand this regression analysis to include data from a wider range of driving experiences, while examining other issues like nuisance effects and driver distraction.

Perhaps the biggest challenge facing the implementation of an FCW system is to prove its effectiveness in reducing crashes and crash injuries. System effectiveness is directly related to crashes avoided, injuries prevented, and lives saved. For example, the effectiveness of an FCW system in avoiding rear-end crashes is defined as follows:

$$
\text { FCW Effectiveness }=\frac{\text { Crashes avoided due to FCW }}{\text { Crashes avoided due to FCW + Number of rear-end crashes }}
$$

To illustrate the meaning of Eq. [1], suppose the crash avoidance effectiveness of an FCW system is $30 \%$. This means that if 100 drivers in cars without an FCW system rear-ended a lead-vehicle, 30 of the collisions would have been avoided ( 70 would still have occurred) had their cars been equipped with an FCW system. FCW system effectiveness may also be expressed in terms of probability. This makes it possible to compute effectiveness estimates and their variances.

$$
\begin{equation*}
\text { Eff }_{\text {FCW }}=\frac{P_{\text {AVOID DUETO FCW }}}{P_{\text {AVOID DUETO FCW }}+P_{\text {CRASH WITH FCW }}}=\frac{P_{\text {CRASH WIHOUT FCW }}-P_{\text {CRASH WITH FCW }}}{P_{\text {CRASH WIHOUT FCW }}} \tag{2}
\end{equation*}
$$

The risk of crashing without an FCW system (the denominator in Eq. [2]) may be estimated by combining naturalistic data - which provides exposure rates for potential crashes - with the General Estimates System (GES). (The GES is maintained by NHTSA and is a national crash database that provides incidence levels of all crashes, including rear-end collisions.) The larger challenge involves estimating how much the probability of crashing changes when an FCW system is in use (the numerator in Eq. [2]).

Probability estimates are substantiated best if they are based on a nationally representative collection of all driving events (such as the GES or the Fatality Analysis Reporting System (FARS), which is a census of all traffic fatalities in the U.S.). One such estimate is found in a recent NHTSA study based on FARS data which reaffirms that threepoint seat belts reduce fatalities by 45 percent in passenger car crashes (Kahane, 2000). Ultimately, the characterization method presented herein may be used to organize and combine all sources of data into a braking event database having a GES-like structure. Then, FCW effectiveness analyses (and other studies) may resemble Kahane's methodology more closely.

## CHARACTERIZATION METHOD

## Theory

The characterization method presented herein is aimed at harmonizing impending rear-end crash data taken from driving simulators and field tests alike. The foundation of the method - first introduced by Burgett and Miller (2001) - is the assumption that vehicle braking may be described by a constant deceleration profile. As such, the deceleration of a vehicle traveling at velocity $\mathrm{V}_{\mathrm{o}}$ is assumed to jump instantaneously from zero to a constant level, $\mathrm{d}_{\mathrm{o}}$, at some brake application time, $\mathrm{t}_{\mathrm{b}}$. Under an impending rear-end crash, a lead-vehicle and following-vehicle are initially separated by a range of $\mathrm{R}_{0}$. Therefore, a set of seven single-valued covariates ( $\mathrm{V}_{\mathrm{Fo}}, \mathrm{d}_{\mathrm{Fo}}, \mathrm{t}_{\mathrm{Fb}}, \mathrm{V}_{\mathrm{L},}, \mathrm{d}_{\mathrm{Lo}}, \mathrm{t}_{\mathrm{Lb}}, \mathrm{R}_{\mathrm{o}}$ ) may be used to describe the actions of both vehicles. (An "L" subscript denotes a lead-vehicle; an "F" denotes a following-vehicle.) In other words, the seven covariates may be used to derive theoretical time-histories that match the experimental ones. Thus, a single data record (rather than several time-histories) may be used to characterize a braking event.

Covariate values are determined using an optimization procedure aimed at finding the best fit to experimental range and velocity data. The process uses a non-linear routine that searches for values of the seven covariates by simultaneously fitting them to velocity and range time-histories. In doing so, the covariates are allowed to take on

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values that are likely to differ slightly from the actual measures. For example, consider the hypothetical velocity time-history curve fit shown in Fig. 2. Actual (naturalistic) deceleration (shown to begin at time $t=0$ in Fig. 2) is always gradual, so the theoretical brake application time (which is assumed to produce an instantaneous "jump" in deceleration) is taken to occur sometime later. This results in an optimized match between the theoretical and actual time-histories of velocity in which the areas under both curves (proportional to the vehicle travel) are nearly equal.


FIGURE 2 Hypothetical curve fit that minimizes the difference between actual and theoretical vehicle travel. Note that $\mathbf{d}^{\text {Theo }}$ is proportional to the slope of $\mathbf{V}^{\text {Theo }}$ after $\mathbf{t}_{\mathbf{b}}{ }^{\text {Theo }}$ and is constant.

The characterization process requires five time-history files as inputs: lead- and following-vehicle velocity, leadand following-vehicle deceleration, and inter-vehicle range. Of these five, a minimum of two must be recorded measurements within the raw data test file. In naturalistic driving tests the lead-vehicle is rarely instrumented. But as long as the following-vehicle speed (usually taken from the vehicle's speedometer cable) and the lead-vehicle range (usually taken from a range-finding device attached to the following-vehicle) are recorded, the other three inputs may be computed numerically.

## Data Sources

The characterization process is demonstrated herein using data from two of the aforementioned sources: the Iowa Driving Simulator (IDS) and the Intelligent Cruise Control (ICC) test. IDS data contain all five inputs within the raw data file. Furthermore, it is rather easy to determine values of the seven covariates such that the characterized fit agrees well with the experimental velocity and range test signals. The simulator is, after all, programmed to produce idealized lead-vehicle kinematics under a contrived scenario that meet the assumptions of the characterization process. Moreover, IDS data are unencumbered by noisy signals and false readings that often infect naturalistic data.

Nonetheless, IDS data provide a good opportunity to test the characterization method even though good-fitting covariates may be found without using it. To test the method fairly, IDS data are treated as naturalistic driving data. Although all five input time-histories are available, the procedure makes use of only the experimental range and following-vehicle velocity data ( $\mathrm{R}^{E x p}$ and $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ ). Moreover, the two time-histories are treated as "noisy" data that need to be digitally filtered. This acts to smooth the data so that deceleration seems more gradual, making it harder to pinpoint the brake application times. Once the covariates are determined, the unused experimental time-histories (lead-vehicle velocity and the accelerations of both vehicles, $\mathrm{V}_{\mathrm{L}}{ }^{E x p}, \mathrm{~d}_{\mathrm{L}}{ }^{E x p}$, and $\mathrm{d}_{\mathrm{F}}{ }^{E x p}$ ) are used to "test" the accuracy of the method by comparing them to the theoretical ones.

The naturalistic ICC data present a more difficult test of the characterization method. ICC braking is more gradual
so that the constant deceleration assumption is always violated to some degree. Therefore, it is more difficult to identify the values of the seven covariates that produce theoretical time-histories that agree with the velocity and range test signals. Other lesser aspects of the ICC data also make the process more difficult. The quality of the ICC range and velocity signals is not as good as IDS data. The resolution of the ICC's $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ signal is just a half-mile per hour and the $\mathrm{R}^{E x p}$ signal is affected by various noise-inducing sources (such as precipitation and road surface roughness). Moreover, the lower sampling rate ( 30 Hz in IDS data versus 10 Hz in ICC data) has a negative effect on characterization process.

## Application

The characterization procedure is outlined below and it makes use of common signal processing algorithms. Digital filtering is accomplished via a Kalman likelihood evaluation (SAS, 2000a). Acceleration inputs are derived by differentiating velocity data using a two-point, backward-looking scheme. The five inputs are denoted by either the "Exp" superscripts (if extracted directly from raw data) or the "Сотр" superscripts (if computed via signal processing). The outputs of the characterization routine are denoted by "Theo" superscripts.

Step 1. Retrieve (or compute) the five input time-histories, $\mathrm{R}^{E x p}, \mathrm{~V}_{\mathrm{F}}{ }^{E x p}, \mathrm{~V}_{\mathrm{L}}{ }^{\text {Comp }}, \mathrm{d}_{\mathrm{F}}{ }^{\text {Comp }}$, and $\mathrm{d}_{\mathrm{L}}{ }^{\text {Comp }}$, as follows:
1.1. Extract the time-histories of range, $\mathrm{R}^{E x p}$, and velocity, $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$, from the raw data.
1.2. Compute the time-histories of velocity, $\mathrm{V}_{\mathrm{L}}{ }^{\text {Comp }}$, and decelerations $\mathrm{d}_{\mathrm{F}}{ }^{\text {Comp }}$ and $\mathrm{d}_{\mathrm{L}}{ }^{\text {Comp }}$ as follows:
1.2.1. Numerically filter $\mathrm{R}^{E x p}$ and $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$.
1.2.2. Compute range rate by differentiating filtered $\mathrm{R}^{E x p}$.
1.2.3. Compute $\mathrm{d}_{\mathrm{F}}^{\text {Comp }}$ by differentiating filtered $\mathrm{V}_{\mathrm{F}}{ }^{\text {Exp }}$.
1.2.4. Compute $\mathrm{V}_{\mathrm{L}}{ }^{\text {Comp }}$ by subtracting range rate (Step 1.2.2) from filtered $\mathrm{V}_{\mathrm{F}}{ }^{\text {Exp }}$.
1.2.5. Compute $\mathrm{d}_{\mathrm{L}}{ }^{\text {Comp }}$ by differentiating $\mathrm{V}_{\mathrm{L}}{ }^{\text {Comp }}$ data.
1.3. Define the starting point and ending point of the event as follows:
1.3.1. Starting point. In most test data, braking events are embedded within a driving file containing several minutes - or even hours - of continuous data. Each event - lasting only about ten seconds must be extracted. In doing so, only data from the braking event itself - and not "normal driving" - are desired. On the other hand, the characterization would be inaccurate if an early portion of the event were truncated from the input data files. Since the beginning of a braking event is usually not well defined, a conservative approach is taken: the beginning is assumed to occur five seconds prior to the onset of following-vehicle braking. This is an arbitrary point and it does effect the outcome of the procedure. It is discussed in more detail later on.
1.3.2. Ending point. This is taken as the time at which the following-vehicle deceleration reaches its maximum, $t_{d F m a x}$. This point is also somewhat arbitrary. In many cases, the braking event is still in progress at $t_{\text {dFmax }}$ and it would seem that the time when the range rate becomes positive (or when the vehicles begin to diverge) would be a better choice for the ending point. The rationale for using $t_{d F m a x}$ is rooted in the form that naturalistic driving data usually take. In naturalistic driving events, peak deceleration often occurs when the following-vehicle driver perceives that the conflict is under control even though the vehicles may continue to converge. Since the characterization procedure is aimed at characterizing "conflicts", the data prior to $t_{d F m a x}$ are of primary interest. Including data beyond $t_{d F \max }$ would generally lower the deceleration estimates and lessen the characterized severity of the "conflict".

Step 2. Express the theoretical time-histories as functions of time, t .
2.1 Write expressions for the times when both vehicles decelerate to a stop, $\mathrm{t}_{\mathrm{Fs}}$ and $\mathrm{t}_{\mathrm{Ls}}$. Then write expressions for the decelerations, $\mathrm{d}_{\mathrm{L}}^{\text {Theo }}$ and $\mathrm{d}_{\mathrm{F}}^{\text {Theo }}$, as functions of t , $\mathrm{t}_{\mathrm{Fs}}$, and $\mathrm{t}_{\mathrm{Ls}}$ :

$$
\begin{align*}
& \mathrm{t}_{\mathrm{Fs}}=\mathrm{V}_{\mathrm{Fo}}{ }^{\text {Theo }} / \mathrm{d}_{\mathrm{Fo}}^{\text {Theo }}+\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }}  \tag{3}\\
& \mathrm{t}_{\mathrm{Ls}}=\mathrm{V}_{\mathrm{Lo}}^{\text {Theo }} / \mathrm{d}_{\mathrm{Lo}}^{\text {Theo }}+\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }}  \tag{4}\\
& \mathrm{d}_{\mathrm{L}}^{\text {Theo }}(\mathrm{t})= \begin{cases}0 ; & \mathrm{t}<\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }} \text { or } \mathrm{t}_{\mathrm{Ls}} \leq \mathrm{t} \\
\mathrm{~d}_{\mathrm{Lo}}^{\text {Theo }} ; & \mathrm{t}_{\mathrm{Lb}}^{\text {Theo }} \leq \mathrm{t}<\mathrm{t}_{\mathrm{Ls}}\end{cases}  \tag{5}\\
& \mathrm{d}_{\mathrm{F}}^{\text {Theo }}(\mathrm{t})= \begin{cases}0 ; & \mathrm{t}<\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }} \text { or } \mathrm{t}_{\mathrm{Fs}} \leq \mathrm{t} \\
\mathrm{~d}_{\mathrm{Fo}}^{\text {Theo }} ; & \mathrm{t}_{\mathrm{Fb}}^{\text {Theo }} \leq \mathrm{t}<\mathrm{t}_{\mathrm{Fs}}\end{cases} \tag{6}
\end{align*}
$$

2.2 Write the expressions for the theoretical time-histories of velocities $\mathrm{V}_{\mathrm{F}}^{\text {Theo }}$ and $\mathrm{V}_{\mathrm{L}}$ Theo , travel distances $\mathrm{X}_{\mathrm{F}}^{\text {Theo }}$ and $X_{L}^{\text {Theo }}$, and range $\mathrm{R}^{\text {Theo }}$ as functions of $\mathrm{t}, \mathrm{t}_{\mathrm{Fs}}, \mathrm{t}_{\mathrm{Ls}}$, and the seven covariates:

$$
\begin{align*}
& \mathrm{V}_{\mathrm{F}}^{\text {Theo }}(\mathrm{t})=\left\{\begin{array}{l}
\mathrm{V}_{\mathrm{Fo}}^{\text {Theo }} ; \mathrm{t}<\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }} \\
\mathrm{V}_{\mathrm{Fo}}^{\text {Theo }}+\mathrm{d}_{\mathrm{Fo}}^{\text {Theo }}\left(\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }}-\mathrm{t}\right) ; \mathrm{t}_{\mathrm{Fs}}>\mathrm{t} \geq \mathrm{t}_{\mathrm{Fb}}^{\text {Theo }} \\
0 ; \mathrm{t} \geq \mathrm{t}_{\mathrm{Fs}}
\end{array}\right.  \tag{7}\\
& \mathrm{V}_{\mathrm{L}}^{\text {Theo }}(\mathrm{t})=\left\{\begin{array}{l}
\mathrm{V}_{\mathrm{Lo}}^{\text {Theo }} ; \mathrm{t}<\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }} \\
\mathrm{V}_{\mathrm{Lo}}^{\text {Theo }}+\mathrm{d}_{\mathrm{Lo}}^{\text {Theo }}\left(\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }}-\mathrm{t}\right) ; \mathrm{t}_{\mathrm{Ls}}>\mathrm{t} \geq \mathrm{t}_{\mathrm{Lb}}^{\text {Theo }} \\
0 ; \mathrm{t} \geq \mathrm{t}_{\mathrm{Ls}}
\end{array}\right.  \tag{8}\\
& \mathrm{X}_{\mathrm{L}}^{\text {Theo }}(\mathrm{t})=\left\{\begin{array}{l}
\mathrm{V}_{\mathrm{Lo}}^{\text {Theo }} \mathrm{t}-0.5 \mathrm{~d}_{\mathrm{L}}^{\text {Theo }}\left(\mathrm{t}-\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }}\right)^{2}+\mathrm{R}_{\mathrm{o}}^{\text {Theo }} ; \mathrm{t} \leq \mathrm{t}_{\mathrm{Ls}} \\
\mathrm{~V}_{\mathrm{Lo}}^{\text {Theo }} \mathrm{t}_{\mathrm{Ls}}-0.5 \mathrm{~d}_{\mathrm{Lo}}^{\text {Theo }}\left(\mathrm{t}_{\mathrm{Ls}}-\mathrm{t}_{\mathrm{Lb}}^{\text {Theo }}\right)^{2}+\mathrm{R}_{\mathrm{o}}^{\text {Theoo }} ; \mathrm{t}_{\mathrm{Ls}}<\mathrm{t}
\end{array}\right.  \tag{9}\\
& \mathrm{X}_{\mathrm{F}}^{\text {Theo }}(\mathrm{t})=\left\{\begin{array}{l}
\mathrm{V}_{\mathrm{Fo}}^{\text {Theo }} \mathrm{t}-0.5 \mathrm{~d}_{\mathrm{F}}^{\text {Theo }}\left(\mathrm{t}-\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }}\right)^{2} ; \mathrm{t} \leq \mathrm{t}_{\mathrm{Fs}} \\
\mathrm{~V}_{\mathrm{Fo}}^{\text {Theo }} \mathrm{t}_{\mathrm{Fs}}-0.5 \mathrm{~d}_{\mathrm{Fo}}^{\text {Theo }}\left(\mathrm{t}_{\mathrm{Fs}}-\mathrm{t}_{\mathrm{Fb}}^{\text {Theo }}\right)^{2} ; \mathrm{t}_{\mathrm{Fs}}<\mathrm{t}
\end{array}\right.  \tag{10}\\
& \mathrm{R}^{\text {Theo }}(\mathrm{t})=\mathrm{X}_{\mathrm{L}}^{\text {Theo }}(\mathrm{t})-\mathrm{X}_{\mathrm{F}}^{\text {Theo }}(\mathrm{t}) \tag{11}
\end{align*}
$$

Step 3. Marquardt's non-linear regression procedure (SAS, 2000b) may be used to find the values of the seven covariates $\left(\mathrm{V}_{\mathrm{Fo}}{ }^{\text {Theo }}, \mathrm{d}_{\mathrm{Fo}}{ }^{\text {Theo }}, \mathrm{t}_{\mathrm{Fb}}^{\text {Theo }}, \mathrm{V}_{\mathrm{Lo}}{ }^{\text {Theo }}, \mathrm{d}_{\mathrm{Lo}}{ }^{\text {Theo }}, \mathrm{t}_{\mathrm{Lb}}^{\text {Theo }}, \mathrm{R}_{\mathrm{o}}{ }^{\text {Theo }}\right.$ ) such that the following expression (which is used as the regression model) is minimized at every point throughout the entire time-history of the event:

$$
\begin{equation*}
\left(\mathrm{R}^{\text {Theo }}-\mathrm{R}^{\text {Exp }}\right)^{2}+\left(\mathrm{V}_{\mathrm{F}}^{\text {Theo }}-\mathrm{V}_{\mathrm{F}}^{\text {Exp }}\right)^{2}+\left(\mathrm{V}_{\mathrm{L}}^{\text {Theo }}-\mathrm{V}_{\mathrm{L}}^{\text {Comp }}\right)^{2} \Rightarrow 0 \tag{12}
\end{equation*}
$$

In Eq. [12], time-histories of $\mathrm{R}^{E x p}$ and $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ are obtained from the actual sensor outputs, while $\mathrm{V}_{\mathrm{L}}{ }^{\text {comp }}$ is derived in Step 1.2. In applying the regression procedure, Marquardt's algorithm requires initial starting points for the covariate estimates. The algorithm is non-deterministic and results will vary depending on the choice of the starting points. Therefore, a deterministic approach is used to select the starting points according to Table 1.

| Covariate | Initial Estimate (starting point for the regression routine) |
| :---: | :---: |
| $\mathrm{d}_{\mathrm{Fo}}{ }^{\text {Theo }}$ | Maximum following-vehicle deceleration of the Step 1.2.3 timehistory (coincides with $\mathrm{t}_{\text {dFmax }}$ ). |
| $\mathrm{d}_{\text {Lo }}{ }^{\text {Theo }}$ | Maximum lead-vehicle deceleration of the Step 1.2.5 time-history. |
| $\mathrm{t}_{\mathrm{Fb}}{ }^{\text {Theo }}$ | Initial brake application time of the following-vehicle as indicated by a discrete time marker, $\mathrm{t}_{\text {Mark }}$, in the raw data file. |
| $\mathrm{t}_{\mathrm{Lb}}{ }^{\text {theo }}$ | Use $\mathrm{t}_{\text {Mark }}$. Since no explicit lead-vehicle brake indicators exist, assume that it coincides with following-vehicle braking. |
| $\mathrm{R}^{\text {theo }}$ | Experimental range, $\mathrm{R}^{\text {Exp }}$, at time $\mathrm{t}_{\text {Mark }}$. |
| $\mathrm{V}_{\mathrm{Fo}}{ }^{\text {Theo }}$ | The following-vehicle velocity, $\mathrm{V}_{\mathrm{F}}{ }^{\text {Exp }}$, at time $=\mathrm{t}_{\text {Mark }}$. |
| $\mathrm{V}_{\text {Lo }}{ }^{\text {Theo }}$ | The lead-vehicle velocity, $\mathrm{V}_{\mathrm{L}}{ }^{\text {comp }}$, at time $=\mathrm{t}_{\text {Mark }}$. |

TABLE 1 Covariates for the Non-Linear Fit Procedure and their Initial Estimates.
There is still no guarantee that these starting points will provide the best fitting covariates. Therefore, a procedure using nested "do loops" is implemented. Initially, the regression routine is run using covariate starting points that are $50 \%$ of their deterministic values. One by one, each covariate is incremented and the regression routine is run. For every run, the sum of the square of the errors (SSE) in the model is computed once the covariates have converged. When all the covariate starting points reach $150 \%$ of their deterministic values, the procedure is terminated. The final covariate values produced by the run having the lowest SSE are taken as the fitted values.

## RESULTS

Figures 3a-3c compare the theoretical time-histories derived from the seven covariates with the experimental timehistories for a typical IDS simulator run. (Thick lines denote theoretical time-histories; thin lines denote the experimental ones.) Recall that $\mathrm{V}_{\mathrm{F}}^{E x p}$ and $\mathrm{R}^{E x p}$ are the only two actual signals that are used in the fit procedure. Unsurprisingly, the theoretical time-histories fit them rather well. More encouraging are the good fits among all the other signals (including the initial brake application times). The good fits attest to the validity of the procedure and give promise to its ability to characterize naturalistic data.

Figures $4 \mathrm{a}-4 \mathrm{c}$ show a set of fitted time-histories of a braking event recorded during the Intelligent Cruise Control test series. Even though the sampling rate is three times lower, the fit is reasonably good up to the point where the driver of the following-vehicle lets off the brake (at about $\mathrm{t}=4$ seconds). But the shortcomings of the characterization method are revealed: the effects of varying deceleration levels - the product of a "brake pump and release" - are not captured as both cars settle down to near-constant speeds after about $t=6$ seconds. The theoretical characterizations incorrectly show both vehicles decelerating until they come to a stop at about $\mathrm{t}=9$ seconds. Nonetheless, the most interesting portion of the interaction - from the time the lead-vehicle first brakes to the point when the vehicles begin to diverge - is captured by the simplified characterization. And in general, during a series of "brake pumps" the most critical interaction occurs at the "pump" having the highest deceleration level (which is usually the first "pump"). Since the time when deceleration is highest marks the end of the fit process, the most interesting portion of the interaction is almost always captured.

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FIGURE 3A IDS simulator range.


FIGURE 3B IDS simulator velocity.


FIGURE 3C IDS simulator deceleration.


FIGURE 4A Naturalistic range.


FIGURE 4B Naturalistic velocity.


FIGURE 4C Naturalistic deceleration.

Regardless of whether the inputs are ICC data or IDS data, the procedure accurately characterizes the event up to the point of closest vehicle approach. This means that the most important portion of a braking event may be described by a single data record (containing seven variables) instead of several time-histories made up of many records (where individual data records are needed for each sample timepoint). Such a reduction in data makes possible a GES-like database structure under which traditional statistical techniques may be applied to compare a large collection of braking events.

## Time-To-Collision Computation

In an impending forward crash, the following-vehicle driver must decelerate to avoid a crash. If not, the followingvehicle will eventually make contact with the lead-vehicle at the time-to-collision (TTC). Thus, the TTC is a projection into the future of current conditions. Throughout the event, an FCW processor may compute TTC in realtime using onboard sensor output to determine if a crash is imminent. A TTC time-history is helpful in understanding how the severity of a conflict changes as an impending crash draws nearer. When researchers examine test data, the TTC is often evaluated at several discrete timepoints throughout the event. In contemplating a database structure where braking events are characterized by a single record, a faithful characterization of time-tocollision is important. A comparison between TTC computed from experimental data and from the fitted covariates follows. But first, the kinematic equations used to compute TTC are given.

## TTC Kinematic Equations

The TTC (denoted as $\mathrm{t}_{\mathrm{cc}}$ in the following equations) may be computed at each point in time by assuming that the prevailing levels of deceleration for both lead- and following-vehicles remain unchanged up to the crash. As such, velocity, deceleration, and range terms on the right-hand side of the following equations represent instantaneous values.

By definition, the positions of both vehicles are the same at $\mathrm{t}_{\mathrm{t}}$ :

$$
\begin{equation*}
\mathrm{X}_{\mathrm{F}}\left(\mathrm{t}_{\mathrm{tc}}\right)=\mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{tc}}\right) \tag{13}
\end{equation*}
$$

At the time of contact, $\mathrm{t}_{\mathrm{t}}$, one of three situations applies:

- The lead-vehicle comes to a stop at precisely the same time that it is struck.
- The lead-vehicle has stopped before it is struck.
- The lead-vehicle is still moving when it is struck.

Consider the first situation. Under a given initial range and initial velocities of both vehicles, there is a theoretical lead-vehicle deceleration level at which the collision takes place precisely at the moment when the lead-vehicle comes to a stop. This deceleration level, denoted as $\mathrm{d}_{\text {Lcross }}$, is a crossover level: if the actual deceleration is greater than $\mathrm{d}_{\text {Lcross }}$, the lead-vehicle stops first. If less than $\mathrm{d}_{\text {Leross }}$, it is still moving at the time of the crash.

When the lead-vehicle decelerates at $\mathrm{d}_{\text {Lrosss }}$, the elapsed time from the beginning of the conflict to the time to collision is denoted as $t_{\text {t-cross }}$. The kinematic equations for the velocities of both vehicles at time $t=t_{t-\text { cross }}$ may be written as:

$$
\begin{align*}
& \mathrm{V}_{\mathrm{F}\left(\mathrm{t}_{\mathrm{c} \text { c-rosss }}\right)}=\mathrm{V}_{\mathrm{F}}-\mathrm{d}_{\mathrm{F}} \mathrm{t}_{\mathrm{tc-cross}} \neq 0  \tag{14}\\
& \mathrm{~V}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{tc} \text { c-ross }}\right)=\mathrm{V}_{\mathrm{L}}-\mathrm{d}_{\mathrm{L} \text { cross }} \mathrm{t}_{\mathrm{tc} \text { c-ross }}=0 \tag{15}
\end{align*}
$$

The time to collision, $\mathrm{t}_{\text {t-cross }}$, may be found from Eq. [15]:

$$
\begin{equation*}
\mathrm{t}_{\mathrm{tc} \text {-cross }}=\mathrm{V}_{\mathrm{L}} / \mathrm{d}_{\mathrm{L} \text { cross }} \tag{16}
\end{equation*}
$$

The kinematic equations for the positions of both vehicles at time $t=t_{\text {c-cross }}$ may be written as:

$$
\begin{align*}
& \mathrm{X}_{\mathrm{F}}\left(\mathrm{t}_{\mathrm{t} \text { c-cross }}\right)=\mathrm{V}_{\mathrm{F}} \mathrm{~V}_{\mathrm{L}} / \mathrm{d}_{\mathrm{Lcross}}-\left(\mathrm{d}_{\mathrm{F}} / 2\right)\left(\mathrm{V}_{\mathrm{L}} / \mathrm{d}_{\mathrm{Lcross}}\right)^{2}  \tag{17}\\
& \mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{t} \text { c-cross }}\right)=\mathrm{R}+\mathrm{V}_{\mathrm{L}}^{2} /\left(2 \mathrm{~d}_{\mathrm{L} \text { cross }}\right) \tag{18}
\end{align*}
$$

The lead-vehicle deceleration, $\mathrm{d}_{\text {Lcross }}$, may be found by equating Eq. [17] to Eq. [18] and solving for $\mathrm{d}_{\text {Lcross }}$ :

$$
\begin{equation*}
\mathrm{d}_{\text {Leross }}=\frac{\mathrm{V}_{\mathrm{F}} \mathrm{~V}_{\mathrm{L}}-.5 \mathrm{~V}_{\mathrm{L}}^{2}+\sqrt{\left(\mathrm{V}_{\mathrm{F}} \mathrm{~V}_{\mathrm{L}}-.5 \mathrm{~V}_{\mathrm{L}}^{2}\right)^{2}-2 \mathrm{R} \mathrm{~d}_{\mathrm{F}} \mathrm{~V}_{\mathrm{L}}^{2}}}{2 \mathrm{R}} \tag{19}
\end{equation*}
$$

The simultaneously occurring "lead-vehicle stop/collision" is unique because it can happen only when $\mathrm{d}_{\mathrm{L}}=\mathrm{d}_{\mathrm{L} \text { cross }}$ for a given set of initial conditions. For all other lead-vehicle deceleration levels (under the same set of initial conditions), one of the other two situations apply:

1. The lead-vehicle comes to a stop before being struck, $\mathrm{d}_{\mathrm{L}}>\mathrm{d}_{\text {Leross }}$.
2. The lead-vehicle is still moving when it is struck, $\mathrm{d}_{\mathrm{L}}<\mathrm{d}_{\text {Lcross }}$.

The following is a derivation of the time-to-collision under these two situations: $\mathrm{t}_{\mathrm{tc} 1}$ for the first situation, $\mathrm{t}_{\mathrm{tc} 2}$ for the second. Consider the first situation in which the lead-vehicle comes to a stop before being struck at time $t_{\mathrm{tcl}}$. This situation is governed by the following three kinematic constraints:

$$
\begin{align*}
& \mathrm{X}_{\mathrm{F}}\left(\mathrm{t}_{\mathrm{cl}}\right)=\mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{sL}}\right)  \tag{20}\\
& \mathrm{t}_{\mathrm{tc} 1}>\mathrm{t}_{\mathrm{sL}}  \tag{21}\\
& \mathrm{~V}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{sL}}\right)=0 \tag{22}
\end{align*}
$$

The kinematic equations for the positions of both vehicles at the time of contact may be written as:

$$
\begin{align*}
& \mathrm{X}_{\mathrm{F}}\left(\mathrm{t}_{\mathrm{cl} 1}\right)=\mathrm{V}_{\mathrm{F}} \mathrm{t}_{\mathrm{cl}}-\left(\mathrm{d}_{\mathrm{F}} / 2\right) \mathrm{t}_{\mathrm{cc} 1}^{2}  \tag{23}\\
& \mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{sL}}\right)=\mathrm{R}+\mathrm{V}_{\mathrm{L}}^{2} /\left(2 \mathrm{~d}_{\mathrm{L}}\right) \tag{24}
\end{align*}
$$

Equating the right hand sides of Eqs. [23] and [24] leads to an expression that may be solved for the time-tocollision, $\mathrm{t}_{\mathrm{cc}}$ (positive root):

$$
\begin{equation*}
\mathrm{t}_{\mathrm{cl} 1}=\frac{\mathrm{V}_{\mathrm{F}}-\sqrt{\mathrm{V}_{\mathrm{F}}-\frac{\left(2 \mathrm{Rd}_{\mathrm{L}}+\mathrm{V}_{\mathrm{L}}^{2}\right) \mathrm{d}_{\mathrm{F}}}{\mathrm{~d}_{\mathrm{L}}}}}{\mathrm{~d}_{\mathrm{F}}} \tag{25}
\end{equation*}
$$

Now consider the second situation where the lead-vehicle is moving when it is struck at time $\mathrm{t}_{\mathrm{t} 2}$. This situation is governed by two kinematic constraints:

$$
\begin{align*}
& \mathrm{X}_{\mathrm{F}}\left(\mathrm{t}_{\mathrm{c} 2}\right)=\mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{c} 2}\right)  \tag{26}\\
& \mathrm{V}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{tc} 2}\right) \neq 0 \tag{27}
\end{align*}
$$

The kinematic equations for the positions of both vehicles at the time of contact may be written as:

$$
\begin{align*}
& X_{F}\left(t_{\mathrm{tc} 2}\right)=V_{\mathrm{F}} \mathrm{t}_{\mathrm{tc} 2}-\left(\mathrm{d}_{\mathrm{F}} / 2\right) \mathrm{t}_{\mathrm{tc} 2}{ }^{2}  \tag{28}\\
& \mathrm{X}_{\mathrm{L}}\left(\mathrm{t}_{\mathrm{t} 2} 2\right)=\mathrm{R}+\mathrm{V}_{\mathrm{L}} \mathrm{t}_{\mathrm{tc} 2}-\left(\mathrm{d}_{\mathrm{L}} / 2\right) \mathrm{t}_{\mathrm{tc} 2}{ }^{2} \tag{29}
\end{align*}
$$

A quadratic expression for $\mathrm{t}_{\mathrm{c} 2}$ is produced by setting Eq. [28] equal to Eq. [29]. The positive root yields the expression for the time-to-collision, $\mathrm{t}_{\mathrm{tc} 2}$ :

$$
\begin{equation*}
\mathrm{t}_{\mathrm{c} 2}=\frac{\left(\mathrm{V}_{\mathrm{L}}-\mathrm{V}_{\mathrm{F}}\right)-\sqrt{\left(\mathrm{V}_{\mathrm{F}}-\mathrm{V}_{\mathrm{L}}\right)^{2}+2 \mathrm{R}\left(\mathrm{~d}_{\mathrm{L}}-\mathrm{d}_{\mathrm{F}}\right)}}{\left(\mathrm{d}_{\mathrm{L}}-\mathrm{d}_{\mathrm{F}}\right)} \tag{30}
\end{equation*}
$$

Using the appropriate equation (Eq. [25] or Eq. [30]), a TTC time-history may be formed by computing the TTC at each point in time. Actually, two time-histories may be formed: an "experimental" one using point-by-point values of $\mathrm{V}_{\mathrm{F}}, \mathrm{V}_{\mathrm{L}}, \mathrm{d}_{\mathrm{F}}, \mathrm{d}_{\mathrm{L}}$, and R in the five input time-histories, and a "theoretical" one using the point-by-point timehistories that are functions of the seven covariates.

## TTC Comparison

For the previous IDS example, Fig. 5 shows the TTC computed from the actual time-histories versus the TTC derived from the fitted covariates. Values for TTC vanish when a collision is not imminent. This happens at the beginning of the event when both vehicles are traveling at constant speeds with the lead-vehicle traveling faster, and later on when the following-vehicle has reached a deceleration level high enough to avoid a crash. Under the characterization process, the vanishing points are instantaneous because deceleration jumps instantaneously. When the actual IDS signals are considered, the first vanishing point is instantaneous, too, because the IDS unit is programmed so that lead-vehicle deceleration jumps instantaneously. The second point, however, is asymptotic and reflects the non-instantaneous braking of the volunteer driver. As with the other IDS fits, the TTC fit is very good.


The TTC's of the previous Intelligent Cruise Control (ICC) example are compared in Fig. 6. The actual vanishing points are not as sharp due to gradual braking in both vehicles. Thus the fit is not as good. But of the "experimental" ICC curves shown in Figs. 4 and 6, recall that only $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ and $\mathrm{R}^{E x p}$ represent recorded sensor signals. All other "experimental" time-histories are derived via numerical procedures. As such, the discrepancies between the actual and theoretical deceleration time-histories are exaggerated. For example, the "experimental" TTC timehistory is derived using signals that have been smoothed by filtering. Therefore, any abrupt decelerations that truly

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did occur are washed out of the "experimental" data in Fig. 4c. The true TTC time-history probably lies somewhere between the two curves shown in Fig. 6.

## DISCUSSION

The characterization procedure, as demonstrated herein, uses the limiting case of having just two test signals, $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ and $\mathrm{R}^{E x p}$, as inputs in order to demonstrate the robustness of the method. In practice, however, the procedure should make use of all signals available. The exact procedure depends on the data. For example, the CAMP test track program - which was run with both vehicles instrumented with accelerometers - provides signals for all timehistory inputs needed. Its characterization process does not require any numerically derived inputs. And although the process is able to fit the IDS events satisfactorily using just $\mathrm{V}_{\mathrm{F}}{ }^{E x p}$ and $\mathrm{R}^{E x p}$ signals, a modified process using all IDS signals (and not any derived time-histories) is even more trustworthy.

There are other ways that the procedure should be modified depending on the type of data. Consider the choice of the time that marks the beginning of a braking event. As explained earlier, this choice ideally excludes normal driving data without truncating a portion of the conflict. Recall that the characterization method assumes a constant pre-brake velocity, and it searches for covariate values that provide the best fit over the entire time-history. The constant velocity assumption is programmed to hold true in IDS events, so they are unaffected by a high proportion of pre-brake data. However, this assumption rarely holds true in naturalistic driving; braking is usually preceded by low-deceleration coasting. If pre-brake coasting data are included, the fit at the end of the event - which is usually the most interesting portion - will suffer.

This brings up another challenge facing the evaluation of an FCW system: how to define the beginning of a conflict. This definition directly affects the observed exposure rates used to determine FCW efficiency in Eq. [2]. As demonstrated herein, the event is arbitrarily chosen to begin 5 seconds prior to following-vehicle braking. In most cases, a large portion of this span covers normal driving; only the later portion is of interest. Alternatively, a measure of vehicle interaction - such as TTC - may be used to trigger the beginning of a conflict. The challenge is to determine a suitable threshold value of TTC (or some other measure) to use as a trigger. Concerns over time-tocollision evoke another modeling option. If a faithful characterization of TTC is critical, then the term (TTC ${ }^{E x p}-$ $\left.\mathrm{TTC}^{\text {Theo }}\right)^{2}$ may be included in the non-linear regression model.

Regardless of how the beginning of an event is determined, the fit problem in naturalistic data may be tempered by introducing a time-dependent scaling factor, $\Delta(\mathrm{t})$, into the non-linear regression model as seen in Eq. [31]. This function is monotonically increasing so that it gives greater weight to the fit at the end (and most interesting portion) of the event.

$$
\begin{equation*}
\Delta(\mathrm{t})\left\{\left(\mathrm{R}^{\text {Theo }}-\mathrm{R}^{\text {Exp }}\right)^{2}+\left(\mathrm{V}_{\mathrm{F}}^{\text {Theo }}-\mathrm{V}_{\mathrm{F}}^{\text {Exp }}\right)^{2}+\left(\mathrm{V}_{\mathrm{L}}^{\text {Theo }}-\mathrm{V}_{\mathrm{L}}^{\text {Compute }}\right)^{2}\right\} \Rightarrow 0 \tag{31}
\end{equation*}
$$

Moreover, a Mean Square Error (MSE, or the SSE divided by the number of timepoints evaluated) may be used to judge whether the characterized fit is satisfactory. If the MSE lies above some threshold value, then the event may be excluded from a statistical analysis since it is not represented accurately by the simplified characterization. Setting the threshold MSE value represents yet another challenge.

Finally, the characterization procedure serves as a diagnostic tool to reveal any errors that may reside in the test signals. Signals may have incorrect units, bias, or phase lag. Such errors will manifest themselves once the characterization process is applied. For example, a time-history of lead-vehicle velocity appears in the raw ICC data file. At first glance, it appears to be consistent with the following-vehicle velocity and range signals. But in fact, it suffers from a slight time lag because the time-history was computed (not measured) using a recursive algorithm. This lag becomes evident once the theoretical velocity (derived from the fitted covariates) is overlaid graphically, as shown in Fig. 7.


FIGURE 7 Time lag in raw ICC lead-velocity data.

## SUMMARY

This paper demonstrates a numerical process for use in characterizing an impending rear-end crash. A numerical iteration procedure is applied to range and velocity test data to determine seven covariates that describe the actions of a following-vehicle and a lead-vehicle. As such, a test event described by several time-history signals may be reduced to a single data record. This opens up the possibility of forming a database of rear-end crash events constructed under a GES-like framework that may be analyzed for such things as driver behavior and warning system effectiveness.

The validity of the characterization procedure is demonstrated using data from a series of tests run on the Iowa Driving Simulator. It is then shown to satisfactorily describe a less-idealistic naturalistic driving event extracted from field test data of an Intelligent Cruise Control system. In subsequent analyses, the covariates are shown to accurately reproduce a variety of test measures, including time-to-collision. As demonstrated, the characterization procedure is carried out with the fewest input signals allowable. Depending on the amount of test data available, the fit procedure may be improved. Guidelines for an improved procedure are provided.

The procedure also serves to check the integrity of the test data. A poor fitting set of covariates may be a symptom of data that are biased, scaled incorrectly, or out of sync.

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