A Simplified Vehicle and Driver Model for Vehicle Systems Development

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Abstract

For the purposes of vehicle systems controller development it is required to have an accurate vehicle model both for virtual simulation of algorithm performance and to develop state and parameter estimator models for use within the system controllers. With the advent of new sensors it will become possible to use full order vehicle models within the controller estimator. Therefore, the purpose behind the work presented in this paper is to develop a parametric vehicle model that can be used for vehicle simulations and within integrated system controllers. An objective of the vehicle model is for it to utilise a straight forward longitudinal and lateral driver model and to use inputs and state variables that can be easily sensed on a production or prototype vehicle.

Introduction

The vehicle model presented in this paper was designed to allow for coupled ride and handling simulation of the vehicle response to driver and road inputs. The 14 degrees of freedom in the model include the translation of the sprung mass z axis road intercept x and y coordinates and the sprung mass vertical position and the three rotational degrees of freedom about the translational axes of the sprung mass. The remaining 8 degrees of freedom are the un-sprung mass (wheel) vertical deflections and their angular rotations. The active degrees of freedom in the vehicle model and their sign convention are summarised in figure 1.

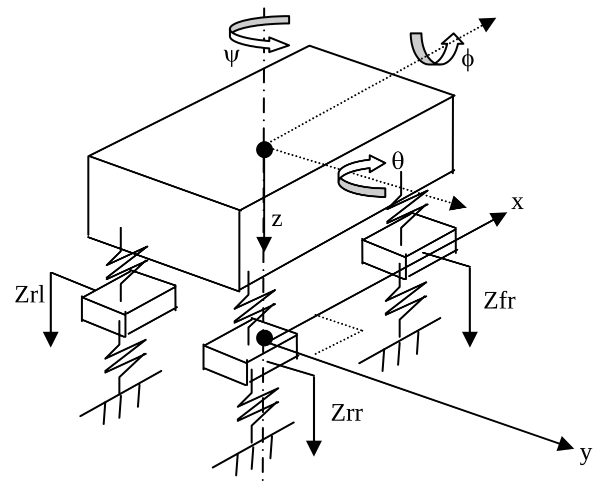

Figure 1: Vehicle local coordinate system and sign convention

The inputs to the model are the wheel steer angles, braking and accelerating wheel torques and the vertical wheel road displacements. The outputs from the vehicle model are the degrees of freedom, their first two derivatives, the longitudinal and lateral tyre slips per wheel and the vertical reactions at each wheel. The equations of motion for the model were derived using the Lagrange approach. In order to simplify the derivation the equations of motion were derived with reference to the local reference frame. This avoids the algebraic complication of expressing the local state variables used for the Lagrange functional as globally referenced variables and hence simplifies the derivation of each Euler-Lagrange equation of motion.

However, the formulation of the equations of motion in the local reference frame implies that a transformation of some of the state variables back into a global reference frame is required for the driver model and inclusion of centripetal cornering forces which otherwise do not arise in the Lagrange formulation. In this work, for planar motion of the vehicle and a track following driver model only road plane x and y degrees of freedom were required to be transformed into the global reference frame. The coordinate transformation can be assumed to be a function of the sprung mass yaw angle alone. The sprung mass rotational degrees of freedom and the un-sprung mass vertical and rotational degrees of freedom were left in the local coordinate reference frame.

In this paper the equation of motion formulation for the vehicle model is presented and the longitudinal and lateral driver models described. The manoeuvre test case presented is for a vehicle negotiating a closed loop sequence of eight 45° turns at a constant speed. Aerodynamic drag in the longitudinal and lateral directions has been included in the simulation as well as sprung mass roll control.

Vehicle model derivation

The vehicle model was derived from first principles using the Lagrange formulation approach where the Lagrange functional was formed for the dynamic system in terms of the generalized coordinates in the local vehicle reference frame [1,2]. The Lagrange functional can be stated as:

$$L(q,\dot{q}) = T(\dot{q}) - U(q)$$

Where $L(q,\dot{q})$ is the Lagrange functional as a function of the generalized coordinates and their derivatives, $T(\dot{q})$ the kinetic energy and $U(q)$ the potential energy. The equations of motion are then formulated by evaluating the Euler-Lagrange equation by differentiation of the Lagrange functional with respect to the generalized coordinates. The Euler-Lagrange functional can be stated as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \frac{\partial F}{\partial \dot{q}} = Q_q$$

Where F represents the non-conservative energy and Q are the non-conservative generalized forces inputting energy into the system (tyre forces etc). Hence, the Lagrange functional can be expressed as a function of the 14 generalized coordinates and their derivatives:

$$L(q,\dot{q}) = L\begin{pmatrix} x, \dot{x}, y, \dot{y}, z, \dot{z}, z_{fl}, \dot{z}_{fl}, z_{fr}, \dot{z}_{fr}, \\ z_{rl}, \dot{z}_{rl}, z_{rr}, \dot{z}_{rr}, \theta_{fl}, \dot{\theta}_{fl}, \theta_{fr}, \dot{\theta}_{fr}, \theta_{rl}, \dot{\theta}_{rl}, \theta_{rr}, \dot{\theta}_{rr} \end{pmatrix}$$

In order to formulate the kinetic energy terms of the Lagrange functional the x and y translation velocities of the sprung mass centre of mass and the four wheels need to be expressed in terms of the road plane z axis intercept x and y component velocities. Therefore, the Lagrangian functional can be expressed entirely in terms of the local vehicle reference frame coordinates. The Lagrangian functional is then repeatedly differentiated with respect to each generalized coordinate for each of the 14 degrees of freedom in turn.

Generalised forces:

The generalised forces are derived using the principle of virtual work where by an expression for the work performed by the system due to an infinitesimal displacement by the external forces is formulated. For this model only the tyre forces were used as the external forces acting on the dynamic system. By assuming the vehicle is in a state of dynamic equilibrium at each time step of the solution the response state variables can be used to formulate the vertical wheel load generalised forces. Note, in the derivation of the generalised forces, for simplicity the suspension parameters [1] that capture the suspension kinematic behaviour were omitted.

Tyre longitudinal and lateral slip:

The lateral tyre slip angles can be derived as functions of the local road plane sprung mass z axis intercept x and y coordinate velocities, yaw rate and wheel steer angles. Similarly the longitudinal wheel slip values in the plane of each wheel are derived from the circumferential speed of the tyre contact patch and the translational speed of the wheel station in the plane of the wheel. Note: The longitudinal and lateral forces are therefore obtained by multiplying the tyre slip values by the corresponding tyre stiffness. For simplicity the lateral and longitudinal tyre stiffness was assumed linear and given constant values in this model.

Centripetal forces:

Because the Lagrangian model derivation detailed above is formulated in the local vehicle reference frame there remains one complication required to make the model accurately represent actual vehicle behaviour. Consider the vehicle negotiating a steady state turn. In vehicle referenced coordinates, as per the equations so far detailed, the result will be that the y coordinate velocity will have a constant value. This is not the case in global coordinates as the vehicle is in fact continuously accelerating laterally towards the centre of rotation. Therefore, in this model the x and y coordinate velocity responses are transformed back into the global reference frame. The centripetal acceleration component of the vehicle towards the centre of rotation can then be evaluated.

The vehicle model can be modified so that a lateral acceleration due to cornering is 'subtracted' from the y generalised force and then 'added' by including a corresponding lateral tyre slip angle on all four wheels, leading to a lateral tyre force. Therefore, the lateral tyre slip due to centripetal acceleration is included in the model but the net lateral acceleration still does not include the centripetal component, hence the locally referenced equations of motion can be used. The calculated centripetal acceleration is also used to augment the sprung mass roll angle generalised force and hence induce body roll during cornering.

Note: The 'internal' tyre slip angles used in the model need to be augmented by the centripetal acceleration induced slip angles for display purposes. These are easily computed from the state variable response and individual tyre lateral stiffness. To take account of the sprung mass centre of gravity not being symmetrically located the centripetal acceleration induced slip angles used internally in the vehicle model and for display purposes need to be factored according the centre of mass position such that the centripetal acceleration induced slip angles do not result in a net yaw moment on the vehicle. It is only the tyre slip angles arising due to translational accelerations that should induce yaw moments for use with the locally referenced vehicle.

Aerodynamic loading:

A simple longitudinal and lateral aerodynamic model has been included in the model in that additional generalised force terms have been added to the x and y equations of motion based on the drag formula given below:

$$Q_w = 1/2\,\rho v^2 C_D A$$

Where C_D is the directional drag coefficient, ρ the air density, A the resistance area and v the velocity magnitude, which for this model will be the locally referenced x and y velocity component response magnitudes.

Roll control:

In the results given in this paper a PID feedback roll control loop was added. The response sprung mass roll angle was subtracted from the demand roll angle (zero for these simulations), factored by the PID gain and fed into the vehicle model as an additional generalised force term in the sprung mass roll angle equations of motion.

Numerical considerations:

It is the case that some of the vehicle model expressions involve terms that have quotients where the denominator is solely a function of one or more of the state variables. Therefore, in order to avoid division by zero and the ensuing numerical singularities, conditional if statements were included in the algorithm to force the quotients to evaluate to zero, as opposed to infinity, under these circumstances.

Vehicle model implementation:

The equations of motion and the supporting expressions, i.e., tyre slip terms, planar to local transformations and centripetal acceleration evaluation, were implemented in a non-linear Simulink model. The Matlab 'linmod' command was then used to linearise the Simulink vehicle model into state space format. A further Simulink model was then created which incorporated the linearised state space model into the vehicle simulation environment. The vehicle simulation environment included the longitudinal and lateral driver model, roll control, aerodynamic drag and the vertical road profile. The whole Simulink simulation was called from a master Matlab file which also defined all the physical parameters required for the vehicle model. The physical parameter set was based on a generic vehicle created with reference to standard texts in the literature [3,4].

Note: the non-linear model is linearised into a state space format to permit the vehicle model to be passed to Simulink S-functions as parameter arguments, thus enabling the use of Matlab script control toolbox functions and bespoke user defined codes in other languages if need be (C, FORTRAN etc) within the S-function script files. This capability is essential if any control algorithms developed using this model are to be converted to real time code for hardware in the loop work.

The results given in this paper were generated by running the simulation using the default Simulink ordinary differential equation 45 Dormond-Prince solver in variable step mode. The default solver used for this simulation quickly converged on a solution (for the results given in this paper the simulation ran for about 30 seconds on a Pentium 4, 1 GHz PC) without numerical instability.

Driver model:

The driver model consists of longitudinal and lateral PID controllers outputting a longitudinal generalised force and an averaged steer angle to the steered wheels.

Longitudinal driver model:

The vehicle model response x velocity component represents the forward speed of the vehicle directly. Therefore, this signal is subtracted from the demand forward speed to give the error signal to feed into the PID controller. The PID controller output is input to the vehicle model as an additional term of the driven wheels generalised force.

Note: The resultant velocity magnitude is evaluated from the locally referenced road plane velocity components. The resulting velocity magnitude signal is then integrated in Simulink to give the absolute distance travelled. The absolute distance travelled evaluated in this way is then used as the indexing variable in look up tables included in the lateral driver model for the demand vehicle track data.

Lateral driver model:

Generally, for driver models used in vehicle simulation software the instantaneous lateral vehicle response is defined in terms of the response radius of curvature and yaw angle [5]. Further, the instantaneous response yaw angle and radius of curvature are derived by relating the vehicle’s current position in the global reference frame to its position as if exactly on the prescribed track but slightly ahead of where it should be based on the absolute distance travelled. In this model the track for the vehicle to follow is defined in terms of the sprung mass yaw angle indexed against the absolute distance travelled. This data can either be generated analytically or measured on an instrumented vehicle. The yaw angle response can be obtained from the driver model. Hence, the yaw angle can be subtracted from the demand yaw angle based on the response absolute distance travelled, and a yaw error signal obtained. Note: steer angle damping is included by feeding back the time integral of the steer angle error signal, factored by a time constant.

Simulation Results

The manoeuvre test case presented is for a vehicle negotiating a closed loop sequence of eight 45° turns at a constant speed. The demand track is run for four different sets of roll, longitudinal speed and lateral driver PID gains. The figures that show state variable results with one trace per plot represent the final simulation for the test case where the ‘best’ PID gains were used to control speed, roll and path following response performance. The ‘best’ PID gains were not selected by any optimal or classical approach but set according to observation of the vehicle model response.

Closed loop turn sequence test case:

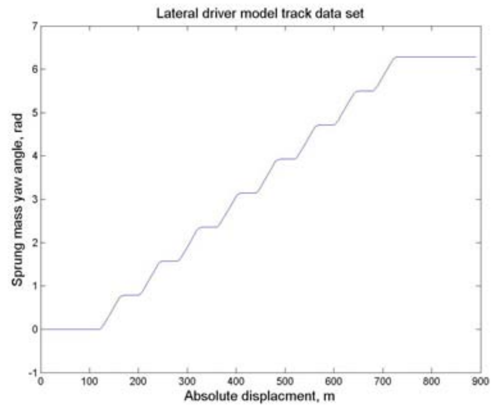

Figure 2: Input yaw angle verses absolute distance travelled data set

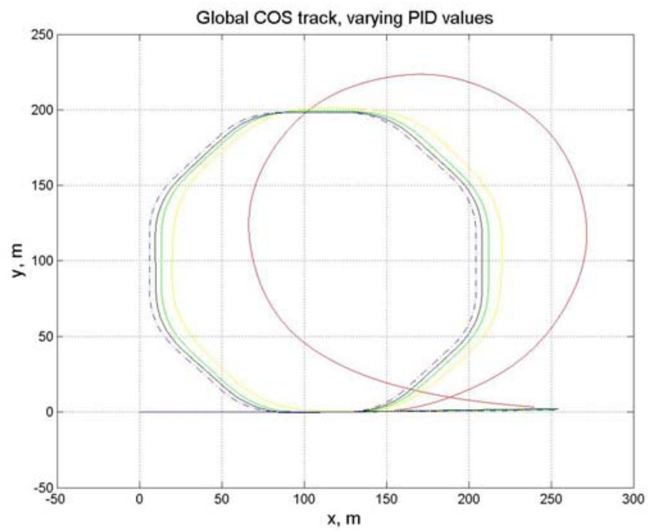

Figure 3: Planar global track (chain dot) and vehicle responses (solid lines)

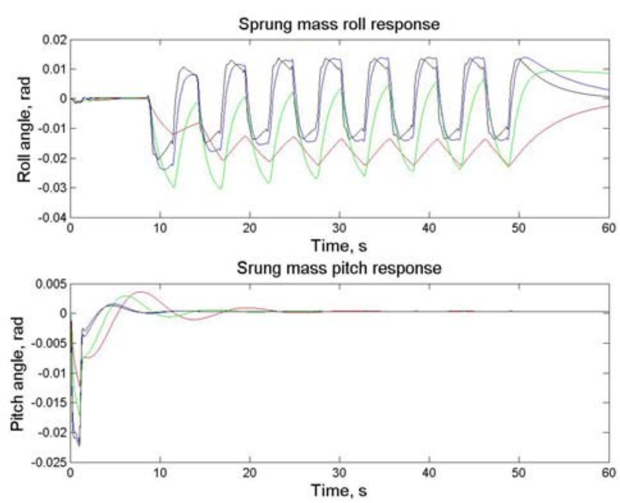

Figure 4: Vehicle sprung mass roll and pitch responses

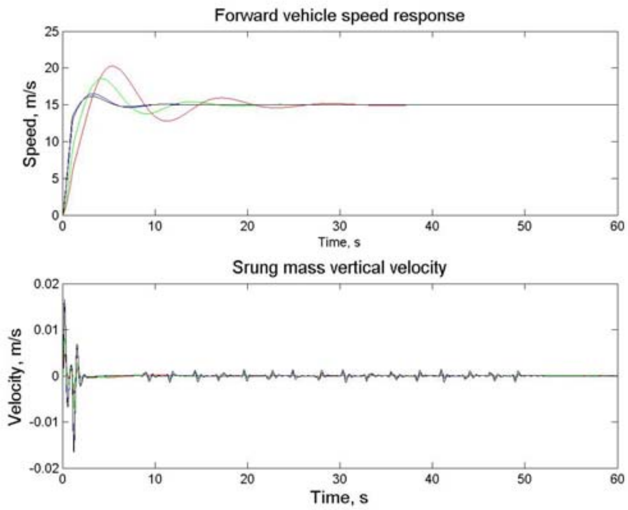

Figure 5: Vehicle forward speed and sprung mass vertical displacement responses

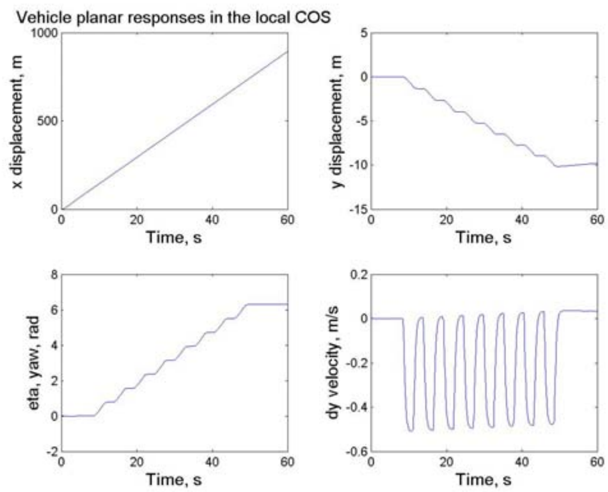

Figure 6: Vehicle yaw and planar response in the local reference frame

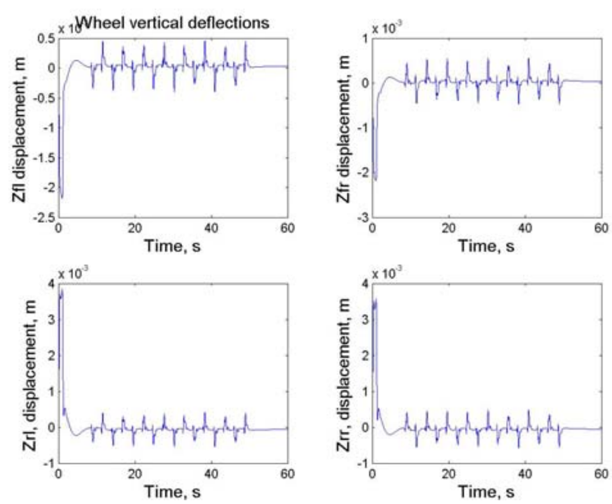

Figure 7: Wheel vertical deflection responses

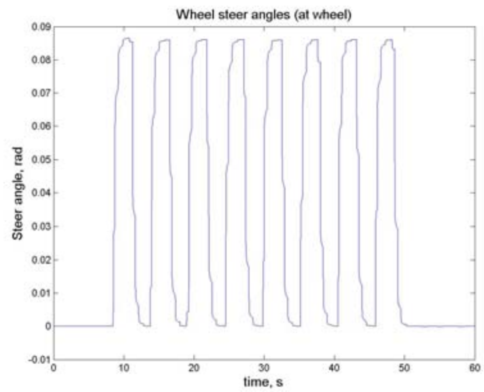

Figure 8: Front wheels steer angle from lateral driver model

Figure 2 shows the input data set for the closed loop turn sequence, which consists of yaw angle verses absolute track position. The effect of the lateral driver gains and time constant value on the ability of the vehicle to follow the demand track are demonstrated in figures 3. Figure 4 shows the roll and pitch responses for the closed loop track for varying roll and forward speed controller gains. The reduced roll response for higher roll controller gains would be significant if a non-linear tyre model was included in the model and the effect of load transfer taken into account. Figure 5 shows the forward speed and sprung mass vertical velocity responses for the closed loop track. As expected, the higher forward speed controller gains result in a quicker rise time and some initial overshoot of the speed response and greater sprung mass deflections.

Figure 6 shows some of the planer local frame of reference responses for the closed loop manoeuvre. Note: the close correlation between the yaw response and the lateral y coordinate displacement. As expected for a near constant forward speed the local reference frame x coordinate displacement builds up linearly with time. Figure 7 shows the vertical deflections of the wheels. As expected the front wheels lift and the rear squat down during acceleration. Figure 8 shows the steer angle on the front wheels as output from the lateral driver controller for the closed loop manoeuvre. Note that the steer angle is the steer angle at the wheel.

Conclusions

A locally referenced 14 DOF combined ride and handling parameterised vehicle model has been derived and validated. Numerical factors have been discussed relating to generation of tyre slip values internally within the vehicle model and the necessity of transforming some of the vehicle model state variables into the global reference frame to account for centripetal effects and generating longitudinal and lateral driver model input. The vehicle state variable responses have been examined for a representative vehicle parameter set and found to agree with intuitively expected vehicle responses. Such a vehicle model approach enables the researcher to have access to the full order model for incorporation into controller simulations for state estimation. The model presented in this paper can be viewed as a basic ‘skeletal’ vehicle model enabling front/rear/all wheel drive, two/four wheel steer and any number of active system control algorithms utilising the state variable set to be simulated. The model can be extended to include suspension kinematic effects by the addition of suspension derivatives in the generalised force expressions within the equations of motion. Non-linear tyre models can also be included in the model. Therefore, the vehicle modelling approach presented in this paper offers the vehicle dynamic systems developer the ability to build vehicle models for inclusion within state and parameter estimator controller algorithms that whilst being far less complex than used with commercial vehicle simulation packages, represent an increase in the order of the vehicle models conventionally built into dynamic vehicle system controllers.

References

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